also applicable for cylindrical coordinates; in this case only the coefficient of p^* has to be changed in the system of equations.

The considerations in the logarithmic region do not directly concern the law of the wall presented here, since new findings in this area can be easily processed mathematically. The authors' suggestions concerning this problem are added to stimulate discussion of the universality of the constants in the logarithmic law of the wall.

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A Generalized Algebraic Stress Transport Hypothesis

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SOME 10 years ago, Rodi[1] and this writer[2,3] independently proposed ways of truncating a second-moment (or Reynolds stress) closure so that the differential transport equations for the Reynolds stresses $\overline{u_iu_j}$ were replaced by a single transport equation for the turbulent kinetic energy k ($\equiv \overline{u_iu_i}/2$) and a set of algebraic equations for the Reynolds stresses. Such closures have become widely used and are commonly known as "algebraic stress models." The form adopted in Refs. 2 and 3 implies that the net transport of $\overline{u_iu_j}$ denoted by T_{ij} (i.e., convection C_{ij} minus diffusion D_{ij}) is related to the turbulent kinetic energy transport T_k by

$$C_{ij} - D_{ij} \equiv T_{ij} = \tfrac{2}{3}\delta_{ij} T_k \tag{1}$$

Mellor and Yamada[4] also obtained Eq. (1) following a different analytical path. Rodi[1] proposed that

$$T_{ij} = \frac{\overline{u_iu_j}}{k} T_k \tag{2}$$

Both forms contract properly and display the correct symmetry in i and j. However, while Eq. (1) gives no transport of the off-diagonal elements (i.e., the shear stresses), Eq. (2) suggests that shear stress transport proceeds just as readily as for the normal stress components. The latter postulate seems more reasonable and has been used by Rodi,[1] Gibson and Launder,[5] and others in the computation of turbulent free shear flows. In these studies the Reynolds stress transport equation is approximated as

$$T_{ij} = P_{ij} - \tfrac{2}{3}\delta_{ij}\epsilon - c_1\epsilon\left(\frac{\overline{u_iu_j}}{k} - \tfrac{2}{3}\delta_{ij}\right) - c_2\left(P_{ij} - \tfrac{1}{3}\delta_{ij}P_{kk}\right) \tag{3}$$

where ϵ is the dissipation rate of turbulence energy and P_{ij} the generation rate of $\overline{u_iu_j}$ by mean shear. A discussion of the physical basis of Eq. (3) is provided in Launder et al.[6]

Elimination of T_{ij} with Eq. (1) or (2) [noting that T_k equals $(\tfrac{1}{2}P_{kk} - \epsilon)$] allows Eq. (3) to be rearranged to an algebraic equation for $\overline{u_iu_j}$. For example, with Eq. (2) we obtain

$$\frac{\overline{u_iu_j} - \tfrac{2}{3}\delta_{ij}k}{k} = \frac{(1-c_2)}{(c_1 - 1 + \lambda)}\left(P_{ij} - \frac{\delta}{3}\, ijP_{kk}\right)\Big/\epsilon \tag{4}$$

where λ denotes $P_{kk}/2\epsilon$ the local ratio of turbulent kinetic energy production to dissipation. For a two-dimensional thin shear flow (with x the flow direction and y the direction of principal gradient), Eq. (4) may be solved to give the following explicit formula for the shear stress

$$-\overline{uv} = \underbrace{\tfrac{2}{3}(1-c_2)\frac{(c_1 - 1 + c_2\lambda)}{(c_1 - 1 + \lambda)^2}\frac{k^2}{\epsilon}}_{c_\mu(\lambda)}\frac{\partial U}{\partial y} \tag{5}$$

a form first given in Ref. 1.

The quantity c_μ in Eq. (5) increases as λ tends toward zero. Figure 1 shows the variation reported in Gibson and Launder[5] where the constant coefficients c_1 and c_2 were given the values 2.2 and 0.55, respectively. The upward trend is, however, less rapid than that exhibited by the experimental data of far wakes and the wakes of self-propelled bodies collected by Rodi[1] whose recommended mean line also appears in Fig. 1. A further unsatisfactory feature of Eq. (5) is that on the axis or plane of symmetry of a jet, λ falls to zero, yet here experiments suggest that a value of c_μ much closer to the local equilibrium ($\lambda = 1$) value is called for.

In considering the cause of these disparities, note first that the off-diagonal Reynolds stresses are predominantly associated with the largest eddies present in the flow and, thus, the turbulent kinetic energy is associated with rather

Table 1 Values of c_μ in round wake and plane jet

Flow	Experiment Min	Experiment Axis	Eq. (8) Min	Eq. (8) Axis	Eq. (5) Min	Eq. (5) Axis
Round wake	0.45	0.56	0.45	0.53	0.2	0.25
Plane jet	0.08	0.12	0.10	0.19	0.10	0.25

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smaller scale motions than the shear stress. Now, the larger the scale of an eddy the longer will be its survival time and thus the greater its contribution to convective transport. We may thus conclude that the approximation for C_{ij} should allow a *preferential* transport of off-diagonal Reynolds stresses. This feature is readily achieved by combining elements of both Eqs. (1) and (2)

$$C_{ij}=C_k\left[(1+\alpha)\left(\overline{u_iu_j}/k\right)-\alpha \tfrac{2}{3}\delta_{ij}\right] \tag{6}$$

where α is a positive coefficient. We adopt a precisely parallel representation of diffusive transport

$$D_{ij}=D_k\left[(1+\beta)\left(\overline{u_iu_j}/k\right)-\beta \tfrac{2}{3}\delta_{ij}\right] \tag{7}$$

Introduction of Eqs. (6) and (7) into Eq. (3) leads to the following implied formula for the coefficient c_μ

$$c_\mu=\tfrac{2}{3}\,\frac{(1-c_2)\left(c_1+(1+\alpha)(\lambda-1)+(\alpha-\beta)\Lambda-(1-c_2)\lambda\right)}{\left(c_1+(1+\alpha)(\lambda-1)+(\alpha-\beta)\Lambda\right)^2} \tag{8}$$

where $\Lambda\equiv D_k/\epsilon$, the ratio of the rates of diffusive gain of turbulence energy to viscous loss.

The choice $\beta=\alpha$ produces the simplest stress-strain relation and, indeed, with these coefficients taken as 0.9 and retaining the above values for c_1 and c_2, the variation of c_μ with λ is very close to the *mean* experimental correlation (Fig. 1). This choice, however, exacerbates the problem of too high viscosities near a jet axis. Now, it must be said that no algebraic approximation is particularly satisfactory for representing diffusive transport—the use of Eq. (7) is justified mainly by the resulting overall success of the ASM formulation. If, therefore, β and α are adjusted to allow Eq. (8) to match as well as possible the experimental behavior in the far round wake and in the jet, β needs to be negative because such a choice helps prevent c_μ from rising excessively on the axis. Table 1 shows the values of c_μ emerging for these two flows with $\alpha=0.3$ and $\beta=-0.8$ with values of λ and Λ taken from experiments cited by Rodi.[1] Clearly Eq. (8) is more successful than Eq. (5) in capturing the observed variation of c_μ in different flows. Further refinement in the choice of α and β must await extensive numerical computations of these flows.

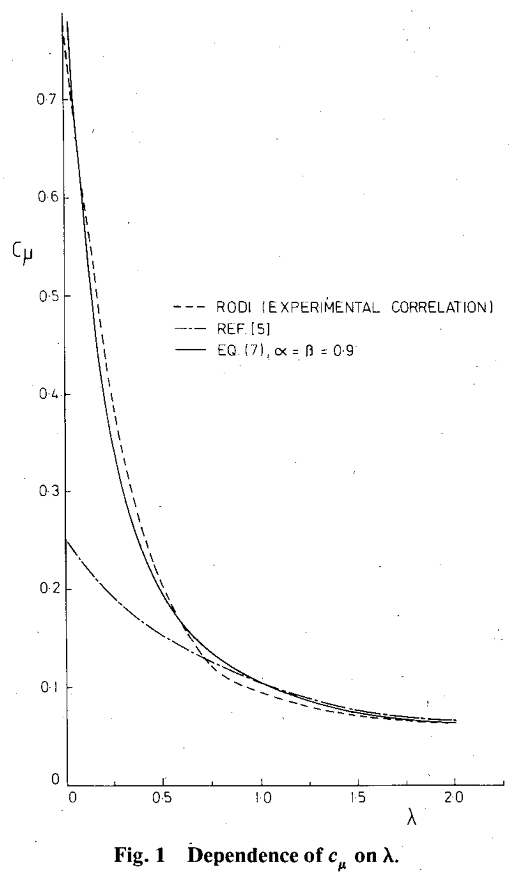

Fig. 1 Dependence of c_μ on λ.

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Effect of Thickness on Airfoil Surface Noise

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Introduction and Discussion of Existing Theory

THE noise radiated from the fixed and rotating surfaces of an aircraft is a matter of concern. The experimental results of this paper can be applied in these areas through appropriate theories describing the noise from an airfoil (e.g., fan, helicopter rotor, flap, and wing noise).

The early airfoil noise theories (e.g., Ref. 1) were for incompressible flow about a thin, small-chord airfoil. The thin airfoil does not disturb the straight mean flow streamlines. Goldstein and Atassi[2] accounted for the real airfoil effects of thickness, camber, and angle of attack for a small-chord airfoil in incompressible flow. These real airfoil effects modify the straight mean flow streamlines which thereby distorts the incident turbulent gusts. They suggest that this distortion will affect the radiated noise, especially at high frequency. Goldstein also formulated fundamental theories that account for the major effects of transverse mean velocity gradients (e.g., jet noise in Ref. 3, and the noise from the leading and trailing edges of a thin infinite plate in Ref. 4). These theories were shown by Olsen to be quite accurate.[5-7] Goldstein also showed (Ref. 1, pp. 137-145) that the early airfoil theory (incompressible flow over a small chord-thin airfoil) is easily modified to include the effect of compressibility. According to Ref. 1, this compressible theory can also be used to predict the spectra at 90 deg (i.e., normal to plane of airfoil) for airfoils of finite chord immersed in a mean velocity gradient. Olsen[8] demonstrated experimentally that this theory predicts the spectra at 90 deg for finite-chord

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